MATHS SAMPLE PAPER

PART-A

Section-I

Section I has 16 questions of 1 mark each.

1. Calculate the largest number which divides 70 and 125, leaves remainders 5 and 8, respectively.

OR

If p is a prime number, then find LCM of p, p^2 and p^3.

2. Explain why 13233343563715 is a composite number?

OR

A number is chosen at random from the numbers — 3, — 2, — 1, 0, 1, 2, 3. What will be the probability that square of this number is less than or equal to 1.

3. How many polynomials can be formed with — 2 and 5 as zeroes?

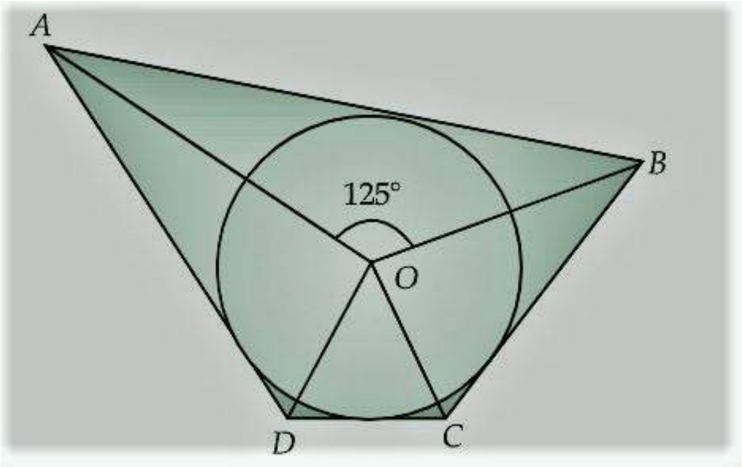
4. Graphically, the pair of equations: $6x - 3y + 10 = 0$, $2x - y + 9 = 0$ Represents what kind of lines.

5. Is the equation $\left(\sqrt{2}x + \sqrt{3}\right)^2 + x^2 = 3x^2 - 5x$ quadratic? Justify.

6. Find the 30th term of the A.P. 10, 7, 4….

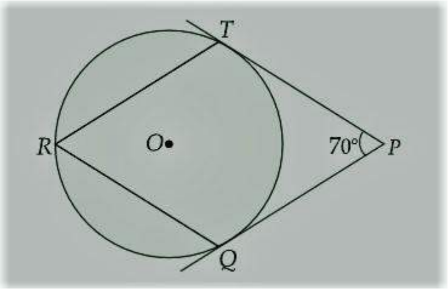
7. Find the distance of the point (— 3, — 4) from the x-axis (in units).

8. In the given figure, if ∠AOB = 125°, then find ∠COD.



OR

In figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If ∠TPQ = 70°, then find ∠TRQ.



9. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after ________________.

10. The perimeter of a sector of a circle of radius 6.5 cm is 29 cm. Its length of arc is ________________.

11. A solid cylinder of radius r and height h is placed over another cylinder of same height and same radius. If h = 2r, total surface area of resulting solid is ________________.

12. If mean and mode of a certain data are 11 and 14 respectively, then its median is ________________.

13. If three different coins are tossed together, then the probability of getting two heads is ________________.

14. If sin (20° + θ) = cos 30°, then find the value of θ.

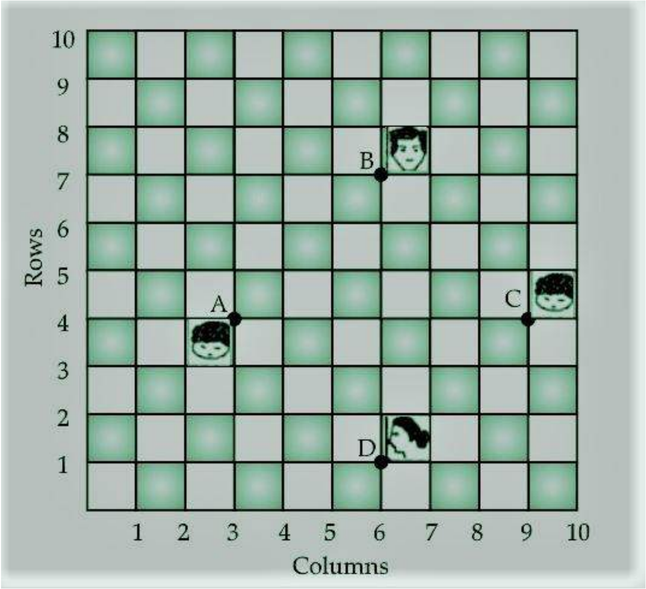
15. If the nth term of an A.P. is pn + q, find its common difference.

16. What is the ratio of the volume of a cube to that of a sphere which will fit inside it?

Section II

Case study based questions are compulsory. Attempt any four sub parts of each question. Each subpart carries 1 mark

17. Case Study based-1: In a room, 4 friends are seated at the points A, B, C and D as shown in figure. Reeta and Meeta walk into the room and after observing for a few minutes Reeta asks Meeta.



(a) What is the position of A ?

(i) (4, 3) (ii) (3, 3) (iii) (3, 4) (iv) None of these

(b) What is the middle position of B and C ?

(i) $\left(\frac{15}{2}, \frac{11}{2}\right)$ (ii) $\left(\frac{2}{15}, \frac{11}{2}\right)$ (iii) $\left(\frac{1}{2}, \frac{1}{2}\right)$ (iv) None of these

(c) What is the position of D ?

(i) (6, 0) (ii) (0, 6) (iii) (6, 1) (iv) (1, 6)

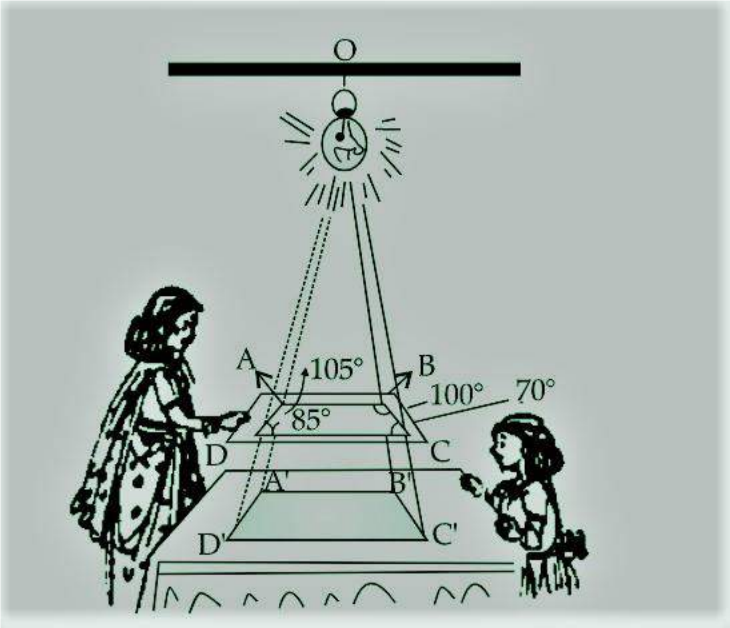
(d) What is the distance between A and B ?

(i) $3\sqrt{2}$ (ii) $2\sqrt{3}$ (iii) $2\sqrt{2}$ (iv) $3\sqrt{3}$

(e) What is the equation of line CD ?

(i) x - y - 5 = 0 (ii) x + y - 5 = 0 (iii) x + y + 5 = 0 (iv) x - y + 5 = 0

18. Case Study based-2 : Seema placed a lightbulb at point 0 on the ceiling and directly below it placed a table. Now, she put a cardboard of shape ABCD between table and lighted bulb. Then a shadow of ABCD is casted on the table as A'B'C'D' (see figure). Quadrilateral A'B'C'D' in an enlargement of ABCD with scale factor 1 : 2, Also, AB = 1.5 cm, BC = 25 cm, CD = 2.4 cm and AD = 2.1 cm; ∠A = 105°, ∠B = 100°, ∠C = 70° and ∠D = 85°.



(a) What is the measurement of angle A′?
(i) 105° (ii) 100° (iii) 70° (iv) 80°

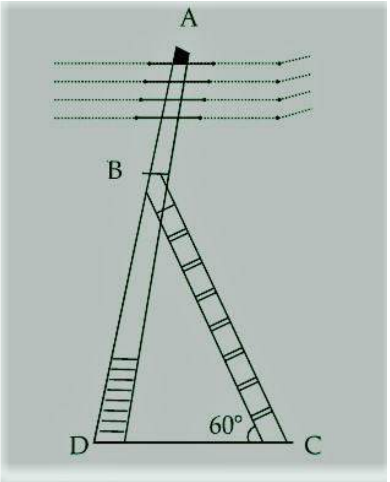
(b) What is the length of A'B'?
(i) 1.5 cm (ii) 3 cm (iii) 5 cm (iv) 2.5 cm

(c) What is the sum of angles of quadrilateral A'B'C'D'?
(i) 180° (ii) 360° (iii) 270° (iv) None of these

(d) What is the ratio of sides A′B′ and A′D′?
(i) 5: 7 (ii) 7: 5 (iii) 1: 1 (iv) 1 : 2

(e) What is the sum of angles of C′ and D′?
(i) 105° (ii) 100° (iii) 155° (iv) 140°

19. Case study based -3: An electrician has to repair an electric fault on the pole of height 5 cm. She needs to reach a point 1.3 m below the top of the pole to undertake the repair work (see figure)



(a) What is the length of BD?
(i) 1.3 m (ii) 5 m (iii) 3.7 m (iv) None of these

(b) What should be the length of Ladder, when inclined at an angle of 60° to the horizontal?

(i) 4.28 m (ii) $\frac{3.7}{\sqrt{3}}$ m (iii) 3.7 m (iv) 7.4 m

(c) How far from the foot of pole should she place the foot of the ladder? (i) 3.7 (ii) 2.14 (iii) $\frac{1}{\sqrt{3}}$ (iv) None of these

(d) If the horizontal angle is changed to 30°, then what should be the length of the ladder?

(i) 7.4 m (ii) 3.7 m (iii) 1.3 m (iv) 5 m

(e) What is the value of $\angle B$?

(i) 60° (ii) 90° (iii) 30° (iv) 180°

20. Case study based -4: $0.0875 = \frac{875}{10^4} = \frac{7}{2^4 \times 5}$

It appears that, we have converted a real number whose decimal expansion terminates into a rational number of the form $\frac{p}{q}$, where p and q are coprime, and the prime factorisation of the denominator (that is, q) has only powers of 2, or powers of 5, or both. We should expect the denominator to look like this, since powers of 10 can only have powers of 2 and 5 as factors. Even though, we have worked only with one example you can say that any real number which has a decimal expansion that terminates can be expressed as a rational number whose denominator is a power of 10. Also, the only prime factors of 10 are 2 and 5. So, cancelling out the common factors between the numerator and the denominator, we find that this real number is a rational number of the form $\frac{p}{q}$, where the prime factorisation of q is of the form $2^n 5^m$, and n, m are some non-negative integers.

(a) Which of the following will have a terminating decimal expansion?

(i) 64/455 (ii) $129/(2^2 \times 5^7 \times 7^5)$ (iii) 24/343 (iv) 84/210

(b) Which of the following will have a non-terminating repeating decimal expansion?

(i) $23/(2^3 \times 5^2)$ (ii) 13/3125 (iii) 39/45 (iv) 17/8

(c) The decimal expansion of $\frac{15}{1600}$ would be

(i) 0.09375

(ii) 0.009375

(iii) 0.09365

(iv) 0.009365

(d) The decimal expansion of $\frac{13487}{6250}$ will terminate after ________ places.

(i) 5 (ii) 3 (iii) 6 (iv) 4

(e) If 0.0514 is expressed in the $\frac{p}{q}$ form, then the sum of the powers m and n of the prime factors of its denominator will be

(i) 5 (ii) 6 (iii) 7 (iv) 8

PART–B

Section III

21. A right circular cylinder and a cone have equal bases and equal heights. If their curved surface areas are in the ratio 8 : 5, show that the ratio between radius of their bases to their height is 3 : 4.

22. Find the sum of deviations of the values 3, 4, 6, 8, 14 from their mean.

23. If A(– 2, 2), B(5, 2) and C(k, 8) are the vertices of a right-angled triangle ABC with ∠B = 90°, then find the value of k.

24. Prove that $\frac{\sin\theta}{1+\cos\theta}+\frac{1+\cos\theta}{\sin\theta}=2cosec\,\theta$.

25. There are 104 students in class X and 96 students in class IX in a school. In an examination, the students are to be evenly seated in parallel rows such that no two adjacent rows are of same class. Find

(i) The maximum number of students of class IX and X in each row.

(ii) The number of parallel rows of each class for the seating arrangement.

26. A bag contains some balls of which x are white, 2x are black and 3x are red. A ball is selected at random. What is the probability that it is (i) not red (ii) white?

Section IV

27. For what values of m and n the following pair of linear equations has infinitely many solutions.

3x + 4y = 12

(m + n) x + 2 (m – n) y = 5m – 1

28. The line segment joining the points A(2, 1) and B(5, – 8) is trisected by the points P and Q, where P is nearer to A. If the point P also lies on the line 2x – y + k = 0, find the value of k.

29. If tan x = n tan y and sin x = m sin y, prove that $cos^2x=\frac{m^2-1}{n^2-1}$.

30. X is a point on the side BC of DABC. XM and XN are drawn parallel to AB and AC respectively meeting AB in N and AC in M. MN produced meets CB produced at T. Prove that TX2 = TB × TC.

31. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60°.

32. The side of a square is 10 cm. Find the area between inscribed and circumscribed circles of the square.

33. Find the value of k for which the quadratic equation

$(k + 1)x^2 - 6(k + 1)x + 3(k + 9) = 0, k \neq -1$ has equal roots.

Section V

34. On a horizontal plane there is a vertical tower with a flag pole on the top of the tower. From a point 9 m away from the foot of the tower, the angles of elevation of the top and foot of the flag pole are 60º and 30º respectively. Find the heights of the tower and the flag pole mounted on it.

35. Find the mean and mode for the following data:

Classes	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	4	8	10	12	10	4	2

36. Prove that in a right-triangle the square of the hypotenuse is equal to the sum of the squares of the other two sides.

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Hints and Solutions

Sample paper

Part – A

Section – I

1. 13 OR p^3.

2. Since the number is divisible by 5, it is a composite number. (It has more than 2 factors) OR 3/7

3. Infinite.

4. Parallel lines (No solution)

5. No. As after solving it does not fit in the standard form of $ax^2 + bx + c = 0$.

6. -77

7. 4 units.

8. $\angle COD = 55°$ OR $\angle TRQ = 55°$

9. 4 places.

10. 16 cm

11. $10\pi r^2$

12. 12

13. 3/8

14. $\theta = 40°$

15. d = p

16. $6:\pi$

Section – II

17. (a) (iii) (3, 4)

(b) (i) $\left(\frac{15}{2}, \frac{11}{2}\right)$

(c) (iii) (6, 1)

(d) (i) $3\sqrt{2}$ units

(e) (i) x - y - 5 = 0

18. (a) ∠A′ = 105°

(b) (ii) 3 cm

(c) (ii) 360°

(d) (i) 5:7

(e) 155°

19. (a) (iii) 3.7 m

(b) (i) 4.28 m (approx)

(c) (ii) 2.14 m (approx.)

(d) (i) 7.4 m

(e) (iii) 30°

20. (a) (iv) 84/210

(b) (iii) 39/45

(c) (ii) 0.009375

(d) (i) 5

(e) (iii) 7

Part – B

Section - III

21. $\frac{2\pi rh}{\pi rl} = \frac{8}{5}$

5h = 4l

$(5h/4)^2 = r^2 + h^2$

h/r = 4/3

The required ratio is 4 : 3

22. Mean = (3 + 4 + 6 + 8 + 14)/5 = 7

Sum of deviations from mean = (-4) + (-3) + (-1) + 1 + 7 = 0

23.

$$AB = \sqrt{(5+2)^2 + (2-2)^2}$$

$$BC = \sqrt{(k-5)^2 + (6)^2} = \sqrt{k^2 + 25 - 10k + 36}$$

$$AC = \sqrt{(k+2)^2 + (8-2)^2} = \sqrt{k^2 + 4 + 4k + 36}$$

By Pythagoras theorem [as $\angle B = 90°$]

$k^2 + 4k + 40 = 49 + k^2 + 61 - 10k$

$14k = 70$

$$k = \frac{70}{14} \Rightarrow 5$$

24.

L.H.S. $$\frac{(\sin\theta)^2 + (1+\cos\theta)^2}{(1+\cos\theta)\sin\theta}$$

$$\Rightarrow \frac{\sin^2\theta + 1 + \cos^2\theta + 2\cos\theta}{(1+\cos\theta)\sin\theta}$$

$$\Rightarrow \frac{2(1+\cos\theta)}{(1+\cos\theta)\sin\theta}$$

$$\Rightarrow 2 \times \frac{1}{\sin\theta}$$

$$\Rightarrow 2\operatorname{cosec}\theta = \text{RHS.}$$

25. (i) Maximum number of students in each row = HCF (104, 96) = 8

(ii) No. of rows for class 10^{th} = 104/8 = 13

No. of rows for class 9^{th} = 96/8 = 12

Total rows = 13 + 12 = 25

26. (i) ½

(ii) 1/6

Section – IV

27.

For infinite solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{3}{m+n} = \frac{4}{2(m-n)} = \frac{-(12)}{-(5m-1)}$$

$$\Rightarrow \quad \frac{3}{m+n} = \frac{2}{m-n}$$

$$\Rightarrow \quad 3m - 3n = 2m + 2n$$

$$\Rightarrow \quad m = 5n \qquad \ldots(i)$$

and $\frac{2}{m-n} = \frac{12}{5m-1} \Rightarrow \frac{1}{m-n} = \frac{6}{5m-1}$

$$5m - 1 = 6m - 6n$$

$$-1 + 6n = m$$

$$-1 + 6n = 5n \qquad \text{[From (i)]}$$

$\therefore \quad n = 1$

and $\quad m = 5.$

28.

P bisects AB in ratio 1 : 2

$\therefore$ Coordinates of P by section formula

$$= \left[\frac{5+4}{1+2}, \frac{-8+2}{3}\right] \Rightarrow [3, -2]$$

Point P lies on $2x - y + k = 0$

$\therefore \quad 2(3) - (-2) + k = 0$

$$k = -8$$

29.

$\tan x = n \tan y$; $\sin x = m \sin y$

$\therefore \quad \cot y = \dfrac{n}{\tan x}$; $\operatorname{cosec} y = \dfrac{m}{\sin x}$

$\operatorname{cosec}^2 y - \cot^2 y = 1$

$\Rightarrow \quad \dfrac{m^2}{\sin^2 x} - \dfrac{n^2}{\tan^2 x} = 1$

$\dfrac{m^2}{\sin^2 x} - \dfrac{n^2 \cos^2 x}{\sin^2 x} = 1$

$m^2 - n^2 \cos^2 x = 1 - \cos^2 x$

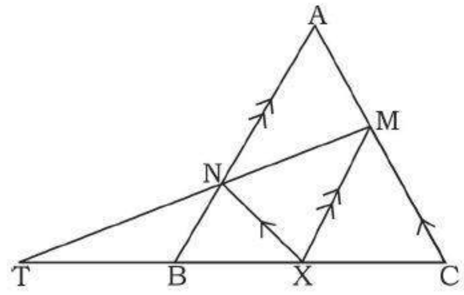
$m^2 - 1 = n^2 \cos^2 x - \cos^2 x$

$m^2 - 1 = \cos^2 x\,(n^2 - 1)$

$cos^2 x = \dfrac{m^2 - 1}{n^2 - 1}$

Hence Proved

30.



Proof : $\Delta TXN \sim \Delta TCM$ [By AA similarty]

$\therefore \quad \dfrac{TX}{TC} = \dfrac{TN}{TM}$ ...(i)

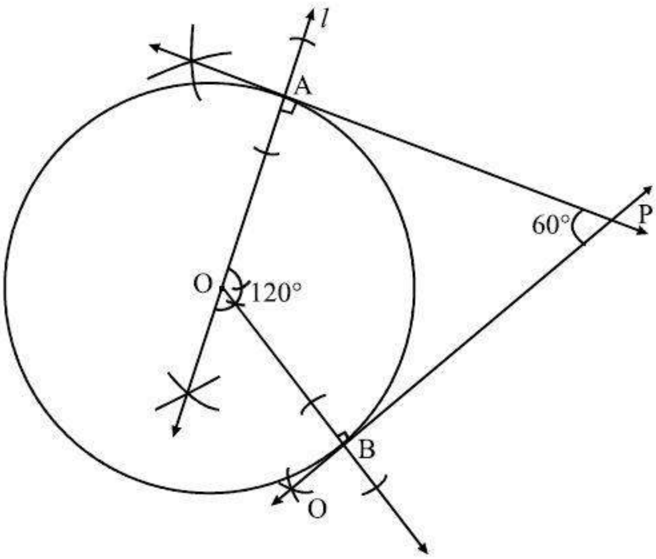
$\Delta TBN \sim \Delta TXM$ [By AA similarty]

$\therefore \quad \dfrac{TB}{TX} = \dfrac{TN}{TM}$ ...(ii)

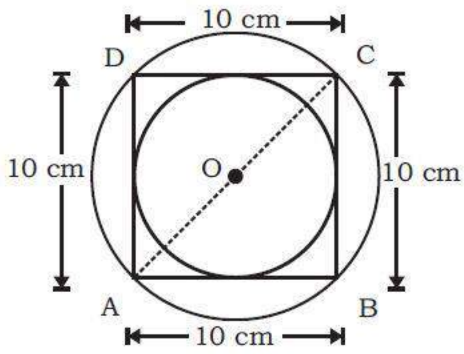
From (i) and (ii); $\dfrac{TX}{TC} = \dfrac{TB}{TX}$

$TX^2 = TB \times TC$

31.



32.



Diameter of inner circle = 10 cm

LABC = 90°

$\therefore$ $AC = 10\sqrt{2}$ cm

Area of inscribed circle = $\pi(5)^2 = 25\pi$ cm²

Area of circumscribed circle = $\pi\left(5\sqrt{2}\right)^2 = 50\pi \text{cm}^2$

Area between two circles = $(50\pi - 25\pi)$ cm²

= 25π cm²

33.

For equal roots; D = 0

$b^2 - 4ac = 0$

$\Rightarrow \quad [-6(k+1)]^2 - 4\,[k+1]3[k+9] = 0$

$\Rightarrow \quad 36\,(k^2 + 1 + 2k) - 12(k+1)\,(k+9) = 0$

$\Rightarrow \quad 3k^2 + 3 + 6k - k^2 - 10k - 9 = 0$

$\Rightarrow \quad 2k^2 - 4k - 6 = 0$

$\Rightarrow \quad k^2 - 2k - 3 = 0$

$\Rightarrow \quad k^2 - 3k + k - 3 = 0$

$\Rightarrow \quad k(k-3) + 1(k-3) = 0$

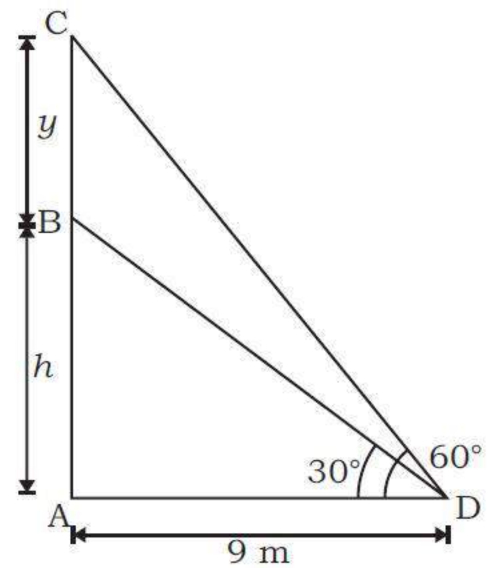
$\Rightarrow \quad (k-3)\,(k+1) = 0$

$\Rightarrow \quad k = 3, -1$

k cannot be -1, $\therefore$ k = 3.

Section – V

34.



Let AB = Height of tower

BC = Height of flag pole

In ΔABD, tan 30° = $\frac{h}{9}$

$$\frac{1}{\sqrt{3}} = \frac{h}{9}$$

$$\frac{9}{\sqrt{3}} = h$$

or $h = 3\sqrt{3}$ m

In ΔACD, tan 60° = $\frac{h+y}{9}$

$$9\sqrt{3} = 3\sqrt{3} + y \Rightarrow y = 6\sqrt{3}m$$

$\therefore$ Height of tower = $3\sqrt{3}$ m

Height of flag pole = $6\sqrt{3}$ m

35.

Class Interval	***Frequency***	x_i	$d_i = x_i - A$	u_i	fiu_i
10 – 20	4	15	–30	–3	–12
20 – 30	8	25	– 20	–2	–16
30 – 40	10	35	–10	–1	–10
40 – 50	12	**45**	0	0	0
50 – 60	10	55	10	1	10
60 – 70	4	65	20	2	8
70 – 80	2	75	30	3	6
	$\Sigma f_i = 50$				$\Sigma f_i u_i = -14$

Mean $(x_i) = A + \frac{\Sigma f_i u_i}{\Sigma f_i} \times h$

$= 45 + \frac{-14}{50} \times 10$

$= 45 - 2.8 \Rightarrow 42.2$

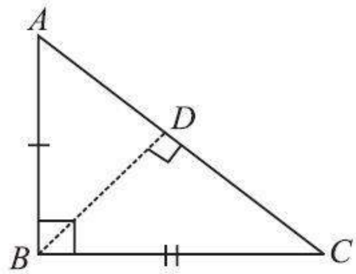
Modal class = 40 – 50; $f_1 = 12$, $f_0 = 10$, $f_2 = 10$

Mode = $l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right] \times h$

$= 40 + \left[\frac{2}{24-20}\right] \times 10$

$= 40 + \frac{2}{4} \times 10 \Rightarrow 45$

36.



Given: A ΔABC in which $\angle ABC = 90^0$.

To prove: $AC^2 = AB^2 + BC^2$.

Construction: Draw $BD \perp AC$.

Proof: In ΔADB and ΔABC, we have

$\angle ADB = \angle ABC$ (each equal to 90^0)

$\angle A = \angle A$ (common)

$\therefore \Delta ADB \sim \Delta ABC$ (AA criterion of similarity)

and so,

$$\frac{AD}{AB} = \frac{AB}{AC} \Rightarrow AD \times AC = AB^2 \quad \text{.... (i)}$$

Now, in ΔBDC and ΔABC, we have

$\angle BDC = \angle ABC$ (each equal to 90^0)

$\angle C = \angle C$ (common)

$\therefore \Delta BDC \sim \Delta ABC$ (AA criterion of similarity)

and so,

$$\frac{DC}{BC} = \frac{BC}{AC} \Rightarrow AC \times DC = BC^2 \quad \text{.... (ii)}$$

Adding (i) and (ii), we get

$AD \times AC + AC \times DC = AB^2 + BC^2$

$(AD + DC) \times AC = AB^2 + BC^2$

$AC \times AC = AB^2 + BC^2$ ($\because AD + DC = AC$)

$\therefore AC^2 = AB^2 + BC^2$

Hence Proved

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